Name:____________________________________ Date:____________________

Math 10/11 Honors HW Challenging Rational Equations Assignment

1

1. If $\dfrac{8}{24} = \dfrac{4}{x+3}$, what is the value of x?

2

If x and y are positive real numbers with $\dfrac{1}{x+y} = \dfrac{1}{x} - \dfrac{1}{y}$, what is the value of $\left(\dfrac{x}{y} + \dfrac{y}{x}\right)^2$?

3

Suppose that x and y are real numbers with $-4 \le x \le -2$ and $2 \le y \le 4$. The greatest possible value of $\dfrac{x+y}{x}$ is

(A) 1 **(B)** -1 **(C)** $-\frac{1}{2}$ **(D)** 0 **(E)** $\frac{1}{2}$

4

Suppose that $a = \dfrac{1}{n}$, where n is a positive integer with $n > 1$.

Which of the following statements is true?

(A) $a < \dfrac{1}{a} < a^2$ **(B)** $a^2 < a < \dfrac{1}{a}$ **(C)** $a < a^2 < \dfrac{1}{a}$

(D) $\dfrac{1}{a} < a < a^2$ **(E)** $\dfrac{1}{a} < a^2 < a$

5

Sylvia chose positive integers a, b and c.

Peter determined the value of $a + \dfrac{b}{c}$ and got an answer of 101.

Paul determined the value of $\dfrac{a}{c} + b$ and got an answer of 68.

Mary determined the value of $\dfrac{a+b}{c}$ and got an answer of k.

The value of k is

(A) 13 **(B)** 168 **(C)** 152 **(D)** 12 **(E)** 169

6

Suppose that c and d are integers with $c > 0$ and $d > 0$ and $\dfrac{2c+1}{2d+1} = \dfrac{1}{17}$. What is the smallest possible value of d?

7

Determine all values of x for which $\dfrac{1}{x^2} + \dfrac{3}{2x^2} = 10$.

8

should she add to change this ratio to 1 : 6?

Suppose that n is a positive integer and that the value of $\dfrac{n^2+n+15}{n}$ is an integer. Determine all possible values of n.

9

a

If $\dfrac{1}{x^2} - \dfrac{1}{x} = 2$, determine all possible values of x.

10

For positive integers a and b, define $f(a,b) = \dfrac{a}{b} + \dfrac{b}{a} + \dfrac{1}{ab}$.
For example, the value of $f(1,2)$ is 3.

(a) Determine the value of $f(2,5)$.

(b) Determine all positive integers a for which $f(a,a)$ is an integer.

(c) If a and b are positive integers and $f(a,b)$ is an integer, prove that $f(a,b)$ must be a multiple of 3.

(d) Determine four pairs of positive integers (a,b), with $2 < a < b$, for which $f(a,b)$ is an integer.

11

(a) Determine the positive integer x for which $\dfrac{1}{4} - \dfrac{1}{x} = \dfrac{1}{6}$.

(b) Determine all pairs of positive integers (a,b) for which $ab - b + a - 1 = 4$.

(c) Determine the number of pairs of positive integers (y,z) for which $\dfrac{1}{y} - \dfrac{1}{z} = \dfrac{1}{12}$.

(d) Prove that, for every prime number p, there are at least two pairs (r,s) of positive integers for which $\dfrac{1}{r} - \dfrac{1}{s} = \dfrac{1}{p^2}$.

12 (a) If $f(x) = \dfrac{x}{x-1}$ for $x \neq 1$, determine all real numbers $r \neq 1$ for which $f(r) = r$.

(b) If $f(x) = \dfrac{x}{x-1}$ for $x \neq 1$, show that $f(f(x)) = x$ for all real numbers $x \neq 1$.

(c) Suppose that k is a real number. Define $g(x) = \dfrac{2x}{x+k}$ for $x \neq -k$. Determine all real values of k for which $g(g(x)) = x$ for every real number x with $x \neq -k$ and $g(x) \neq -k$.

(d) Suppose that a, b and c are non-zero real numbers. Define $h(x) = \dfrac{ax+b}{bx+c}$ for $x \neq -\dfrac{c}{b}$. Determine all triples (a, b, c) for which $h(h(x)) = x$ for every real number x with $x \neq -\dfrac{c}{b}$ and $h(x) \neq -\dfrac{c}{b}$.

13 Let $P(x) = x^2 - 3x - 7$, and let $Q(x)$ and $R(x)$ be two quadratic polynomials also with the coefficient of x^2 equal to 1. David computes each of the three sums $P+Q$, $P+R$, and $Q+R$ and is surprised to find that each pair of these sums has a common root, and these three common roots are distinct. If $Q(0) = 2$, then $R(0) = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

14 AIME 2022

There is a polynomial $P(x)$ with integer coefficients such that

$$P(x) = \frac{(x^{2310}-1)^6}{(x^{105}-1)(x^{70}-1)(x^{42}-1)(x^{30}-1)}$$

holds for every $0 < x < 1$. Find the coefficient of x^{2022} in $P(x)$.

15 AIME 2014

Let m be the largest real solution to the equation

$$\frac{3}{x-3}+\frac{5}{x-5}+\frac{17}{x-17}+\frac{19}{x-19}=x^2-11x-4$$

There are positive integers a, b, and c such that $m=a+\sqrt{b+\sqrt{c}}$. Find $a+b+c$.

16 AIME

Let x and y be real numbers satisfying $x^4y^5+y^4x^5=810$ and $x^3y^6+y^3x^6=945$. Evaluate $2x^3+(xy)^3+2y^3$.

17 AIME2016

Let $P(x)$ be a nonzero polynomial such that $(x-1)P(x+1)=(x+2)P(x)$ for every real x, and $\left(P(2)\right)^2=P(3)$. Then $P(\tfrac{7}{2})=\tfrac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.